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Antenna characteristics: definitions

Antennas

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Wave equation for potentials

Vector and scalar potentials

$$\mu \boldsymbol{H}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) \quad ; \quad \boldsymbol{E}(\boldsymbol{r}) = -j\omega \boldsymbol{A}(\boldsymbol{r}) - \nabla V$$

Linked by Lorentz' gage (other gages are possible, but this one is very convenient)

$$\nabla \cdot \boldsymbol{A}(\boldsymbol{r}) + j\omega\mu\varepsilon V = 0$$

Thus

$$\boldsymbol{E}(\boldsymbol{r}) = -j\omega \boldsymbol{A}(\boldsymbol{r}) + \frac{\nabla(\nabla \boldsymbol{A}(\boldsymbol{r}))}{j\omega\varepsilon\mu}$$

Wave equation for potentials

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+j\omega\varepsilon\boldsymbol{E}$$

$$\nabla\times\frac{1}{\mu}\nabla\times\boldsymbol{A}=\boldsymbol{J}+j\omega\varepsilon\boldsymbol{E}$$

$$\nabla\times\nabla\times\boldsymbol{A}=\mu\boldsymbol{J}+j\omega\varepsilon\mu\boldsymbol{E}$$

$$\nabla\left(\nabla\cdot\boldsymbol{A}\right)-\nabla^2\boldsymbol{A}=\mu\boldsymbol{J}+j\omega\varepsilon\mu\left(-j\omega\boldsymbol{A}+\frac{\nabla\left(\nabla\cdot\boldsymbol{A}\right)}{j\omega\varepsilon\mu}\right)$$

$$\nabla^2\boldsymbol{A}+\omega^2\varepsilon\mu\boldsymbol{A}=-\mu\boldsymbol{J}$$

$$\left(\nabla^2+k^2\right)\boldsymbol{A}=-\mu\boldsymbol{J}$$

We obtain is a similar way $\left(\nabla^2+k^2\right)V=-\frac{\rho}{\varepsilon}$

Wave equation for potentials

Vector potential is linked to currents, scalar potential

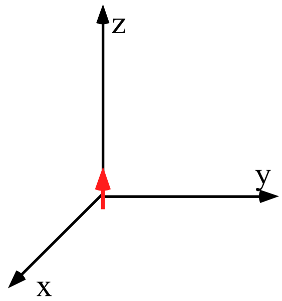
Propagation velocity is given by $c=\frac{\omega}{k}=\frac{1}{\sqrt{\varepsilon\mu}}$

In free space: $c=c_0=299800\ km/s\approx 3\cdot 10^8\ m/s$

And the wavelength is given by $\lambda=\frac{2\pi}{k}=\frac{c}{f}$

Solution for elementary sources

Let us consider an elementary source (Hertzian dipole) Aligned along the z axis



The corresponding vector potential is obtained solving

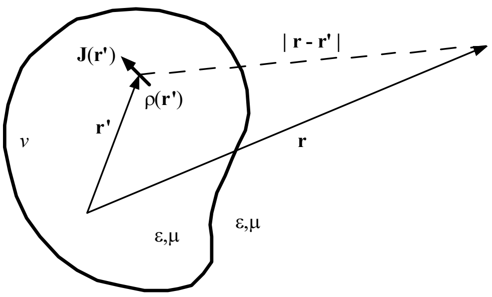
$$\left(\nabla^2 + k^2\right)\boldsymbol{A} = -\mu\delta(\boldsymbol{r})\hat{\boldsymbol{z}}$$

We get $$\boldsymbol{A}(\boldsymbol{r}) = \hat{\boldsymbol{z}}\frac{\mu}{4\pi}\frac{e^{-jkr}}{r}$$

In the same way, we obtain for the scalar potential of A charge located at the origin

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon}\frac{e^{-jkr}}{r}$$

Solutions for any sources



Consider a volume v containing a source distribution **J**(**r**) and a charge distribution ρ(**r**). To compute the potentials due to these sources, we perform integrate the solution for the Hertzian dipole over the volume v in spherical coordinates.

$$\boldsymbol{A}(r) = \frac{\mu}{4\pi}\int_v dv'\boldsymbol{J}(\boldsymbol{r}')\frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$V(\boldsymbol{r}) = \frac{1}{4\varepsilon}\int_v dv'\rho(\boldsymbol{r}')\frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

Findings

- The Solutions for the potentials are a superposition of spherical waves
- The wave number and wavelength are given by

$$k = \omega\sqrt{\varepsilon\mu} \;\; ; \;\; \lambda = \frac{2\pi}{k}$$

- The results are valid for any distance **r**
- Le fields are obtained using

$$\mu \boldsymbol{H}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) \quad ; \quad \boldsymbol{E}(\boldsymbol{r}) = -j\omega \boldsymbol{A}(\boldsymbol{r}) - \nabla V$$

Special case: antennas

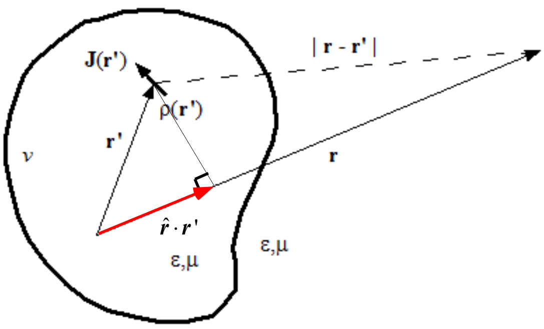
In the case of antenna radiation, we are interested in what happens far from the antenna, in the so called ***far field*** region. Thus, when ***r*** is much larger than the wavelength and the dimensions of the antenna, we can replace |***r-r'***| by a series expansion. For the amplitude

It is enough to consider

$$|\boldsymbol{r} - \boldsymbol{r}'| \approx r$$

For the phase, we need more precision:

$$|\boldsymbol{r} - \boldsymbol{r}'| \approx r - \hat{\boldsymbol{r}} \cdot \boldsymbol{r}'$$
$$= r - x'\sin\theta\cos\varphi - y'\sin\theta\sin\varphi - z'\cos\theta$$



Special case: antennas

We obtain finallly:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_v dv'\boldsymbol{J}(\boldsymbol{r}')\frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|} = \frac{\mu}{4\pi}\int_v dv'\boldsymbol{J}(\boldsymbol{r}')\frac{e^{-jkr}e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}}{r} = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\boldsymbol{f}(\theta,\varphi)$$

$$\boldsymbol{f}(\theta,\varphi) = \int_v dv'\boldsymbol{J}(\boldsymbol{r}')e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}$$

Where ***f*** is called the vector integral, and plays an important role in antenna theory.

Antennas

Radiated fields

Considering the phase and amplitude approximations for |***r-r'***|, we get:

$$\boldsymbol{H} = \frac{j\omega}{Z_c}\boldsymbol{A}\times\hat{\boldsymbol{r}}$$

$$\boldsymbol{E} = j\omega\left[\boldsymbol{A}-\hat{\boldsymbol{r}}(\hat{\boldsymbol{r}}\cdot\boldsymbol{A})\right]$$

$$\boldsymbol{E} = j\omega\hat{\boldsymbol{r}}\times(\hat{\boldsymbol{r}}\times\boldsymbol{A})$$

Moreover, it can easily be shown that

$$\boldsymbol{H} = \frac{1}{Z_c}\hat{\boldsymbol{r}}\times\boldsymbol{E}$$

$$\boldsymbol{E} = Z_c\hat{\boldsymbol{r}}\times\boldsymbol{H}$$

Antennas

Radiated fields

In spherical coordinated, this becomes

$$E_\theta = -j\omega A_\theta \;;\; E_\varphi = -j\omega A_\varphi \;;\; E_r = 0$$

$$H_\theta = \frac{E_\varphi}{Z_c} \;;\; H_\varphi = \frac{E_\theta}{Z_c} \;;\; H_r = 0$$

Or in terms of the vector integral $\boldsymbol{f}$

$$E_\theta(r,\theta,\varphi) = -\frac{jZ_c}{2\lambda}\frac{e^{jkr}}{r}\hat{\boldsymbol{\theta}}\cdot\boldsymbol{f}(\theta,\varphi)$$

$$E_\varphi(r,\theta,\varphi) = -\frac{jZ_c}{2\lambda}\frac{e^{jkr}}{r}\hat{\varphi}\cdot\boldsymbol{f}(\theta,\varphi)$$

The Poynting vector is given by

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{E}\times\boldsymbol{H}^* = \frac{1}{2Z_c}|\boldsymbol{E}|^2\,\hat{\boldsymbol{r}}$$

The radiated power density

From the purely radial Poynting vector, we obain the outward oriented power density:

$$p(r,\theta,\varphi) = \boldsymbol{S}\cdot\boldsymbol{e}_r = \frac{1}{2Z_c}|\boldsymbol{E}|^2 = \frac{1}{2Z_c}\left(|E_\theta|^2 + |E_\varphi|^2\right)$$

Which can also be expressed in terms of the vector integral

$$p(r,\theta,\varphi) = \frac{Z_c}{8\lambda^2}\frac{1}{r^2}\left(|f_\theta|^2 + |f_\varphi|^2\right) \quad [W/m^2]$$

The radiation intensity, independent of *r,* is given by

$$U(\theta,\varphi) = r^2 p(r,\theta,\varphi) = \frac{Z_c}{8\lambda^2}\left(|f_\theta|^2 + |f_\varphi|^2\right) \quad [W/steradian]$$

This intensity corresponds to a power per solid angle

Summary

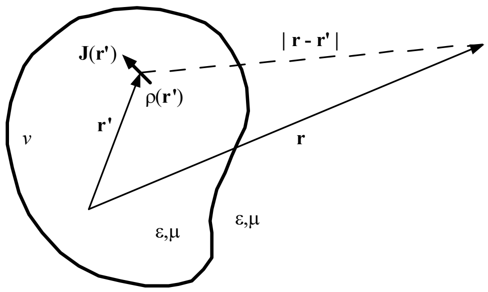
In the far field, we have :

$$E_\theta(r,\theta,\varphi) = -\frac{jZ_c}{2\lambda}\frac{e^{jkr}}{r}\hat{\boldsymbol{\theta}}\cdot \boldsymbol{f}(\theta,\varphi) \;;\; E_\varphi(r,\theta,\varphi) = -\frac{jZ_c}{2\lambda}\frac{e^{jkr}}{r}\hat{\boldsymbol{\varphi}}\cdot \boldsymbol{f}(\theta,\varphi)$$

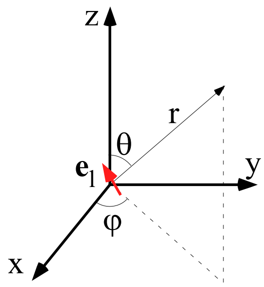
$$H_\theta = -E_\varphi / Zc \quad ; \quad H_\varphi = E_\theta / Zc \quad ; \quad E_r = H_r = 0$$

with

$$\boldsymbol{f}(\theta,\varphi) = \int_v dv' \boldsymbol{J}(\boldsymbol{r}') e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}$$



Example: The Hertzian dipole



The Hertzian dipole is a short filament of current of length *Δl* Supporting a current *I*

$$I(t) = \sqrt{2} I \cos(\omega t)$$

Limit: *Δl=dl*

In practice, this means that Δl is much shorter than the wavelength and that I is invariant on the filament

Example: The Hertzian dipole

In the case of a Hertzian dipole oriented along a random orientation $\hat{\imath}$, and placed at the origin, the current density is given by

$$\boldsymbol{J} = \frac{\hat{\boldsymbol{\imath}} I}{\Delta s}$$

And the elementary volume by $dv' = \Delta s \, dl'$

The vector integral is given by $\boldsymbol{f}(\theta,\varphi) = \hat{\boldsymbol{\imath}} \int_0^{\Delta l} dl' I = I \Delta l \hat{\boldsymbol{\imath}}$

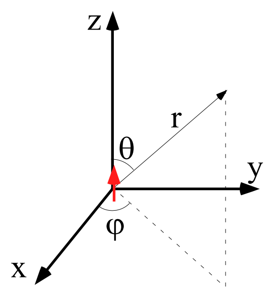
And the vector potential by $\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu I \Delta l}{4\pi} \frac{e^{-jkr}}{r} \hat{\boldsymbol{\imath}}$

The electric and magnetic fields are given by

$$\boldsymbol{E}(r,\theta,\varphi) = \frac{jZ_c}{2\lambda} I \Delta l \frac{e^{-jkr}}{r} \hat{\boldsymbol{r}} \times \left(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{\imath}}\right)$$

$$\boldsymbol{H}(r,\theta,\varphi) = \frac{j}{2\lambda} I \Delta l \frac{e^{-jkr}}{r} \left(\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{r}}\right)$$

Example: A Hertzain dipole oriented along the z axis



$$E_\theta(r,\theta,\varphi) = \frac{jZ_c}{2\lambda} I \Delta l \frac{e^{-jkr}}{r} \sin\theta$$

$$E_\varphi(r,\theta,\varphi) = 0$$

$$H_\theta(r,\theta,\varphi) = 0$$

$$H_\varphi(r,\theta,\varphi) = \frac{j}{2\lambda} I \Delta l \frac{e^{-jkr}}{r} \sin\theta$$

$$U(\theta,\varphi) = \frac{Z_c}{8\lambda^2} (I \Delta l)^2 \sin^2\theta$$

$$p(r,\theta,\varphi) = \frac{1}{r^2} \frac{Z_c}{8\lambda^2} (I \Delta l)^2 \sin^2\theta$$

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Radiation pattern

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Field pattern

Graphic representation of f_θ (E_θ and H_φ) and f_φ(E_φ and H_θ)

There is one pattern for the amplitude and one for the phase

Normalized representation of the amplitude

$$D_{E\theta}(\theta,\varphi) = | E_\theta(\theta,\varphi) | / | E_\theta(\theta_{max},\varphi_{max}) | = | f_\theta(\theta,\varphi) | / | f_\theta(\theta_{max},\varphi_{max}) |$$

$$D_{E\varphi}(\theta,\varphi) = | E_\varphi(\theta,\varphi) | / | E_\varphi(\theta_{max},\varphi_{max}) | = | f_\varphi(\theta,\varphi) | / | f_\varphi(\theta_{max},\varphi_{max}) |$$

In dB : $D_{E\varphi,\theta}$ (dB) = 20 log10 $|D_{E\varphi,\theta}|$

Caution: $\theta_{max}, \varphi_{max}$ can be different for the θ and φ components !

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Normalization choice

- Each component individually with respects to its own max.
- The two componts with respect to the maximum maximorum

$$\max(|E_\theta|_{\max},\ |E_\varphi|_{\max})$$

- The two compoonts with respect to the max. of the vector amplitude

$$\sqrt{\boldsymbol{E}\cdot\boldsymbol{E}^*}_{\max} = \max\left[\sqrt{E_\theta E_\theta^* + E_\varphi E_\varphi^*}\right]$$

Power pattern

Graphic represntation of $p(r,\ \theta,\varphi)$ or $U(\theta,\varphi) = r^2 p$

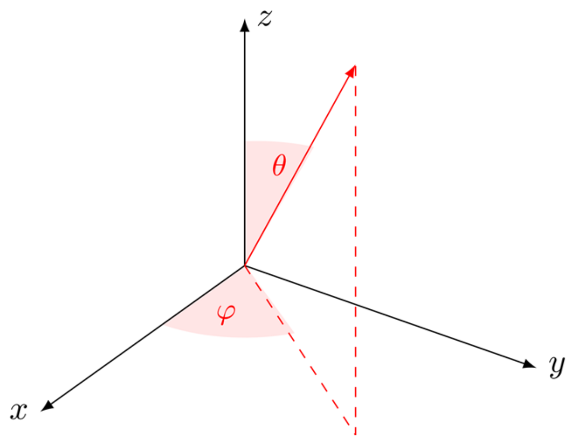
In the far field, this is a real scalar

$D_P(\theta,\varphi) = p(r,\theta,\varphi)\ / p\ (r,\theta_{max},\varphi_{max}) = U(\theta,\varphi)/U\ (\theta_{max},\varphi_{max})$

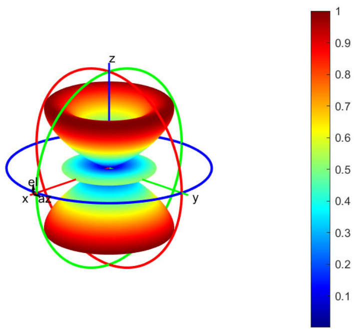
D_P (dB) = 10 $\log_{10} |D_P|$

Note : p is proportional to$(|E_\theta|^2 + |E_\varphi|^2)$. Thus, if f_θ or $f_\varphi = 0$
D_p(dB) = $D_{\theta,\varphi}$(dB)

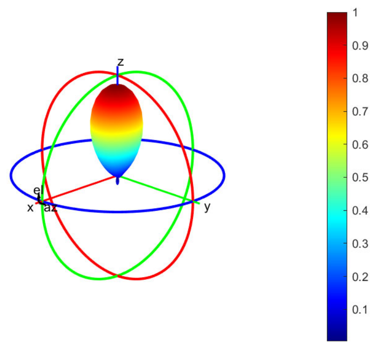
Coordinate system



3-D Polar (linear patterns)

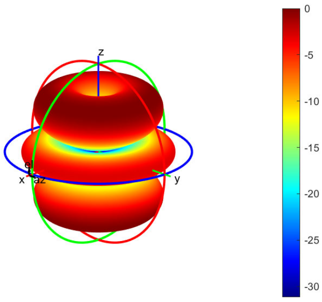


1.5λ dipole

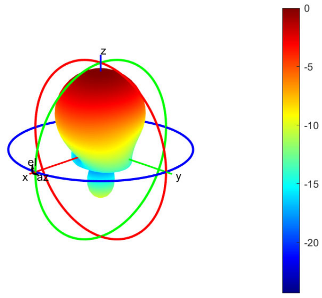


Horn antenna

3-D Polar (patterns in dB)

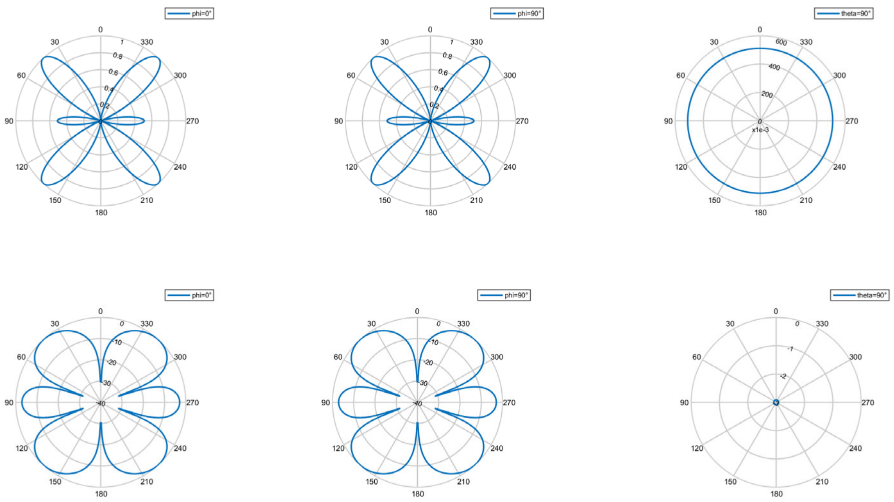


1.5λ dipole

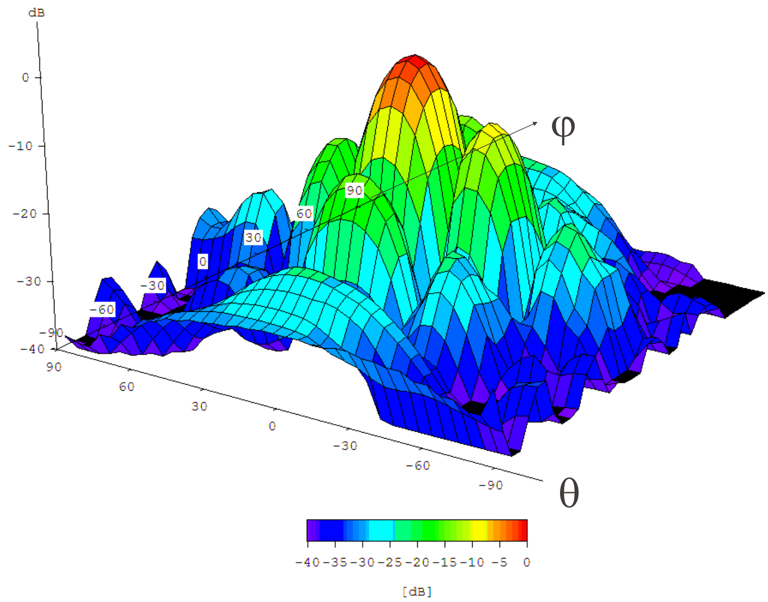


Horn antenna

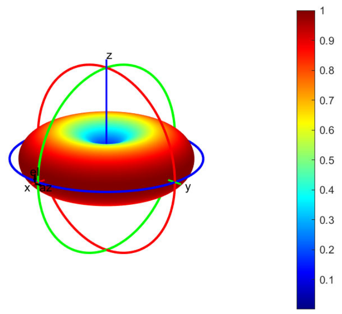
Polar patterns: cuts (1.5 λ dipole)

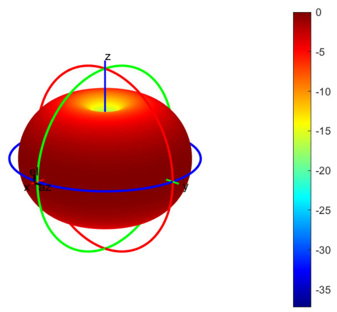


3-D carthesian (in DB)



Half wave dipole : 3D polar pattern





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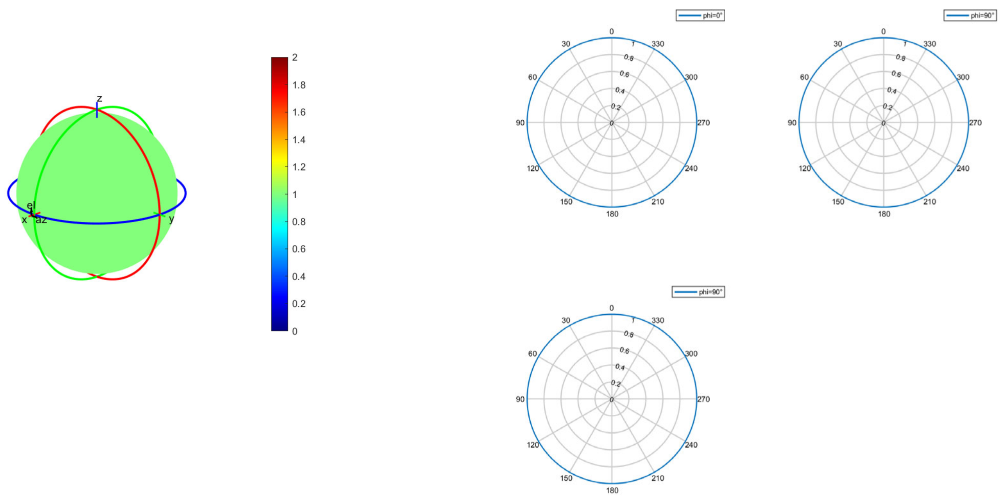
Example : Isotropic antenna (does not exist, but is useful in theory)



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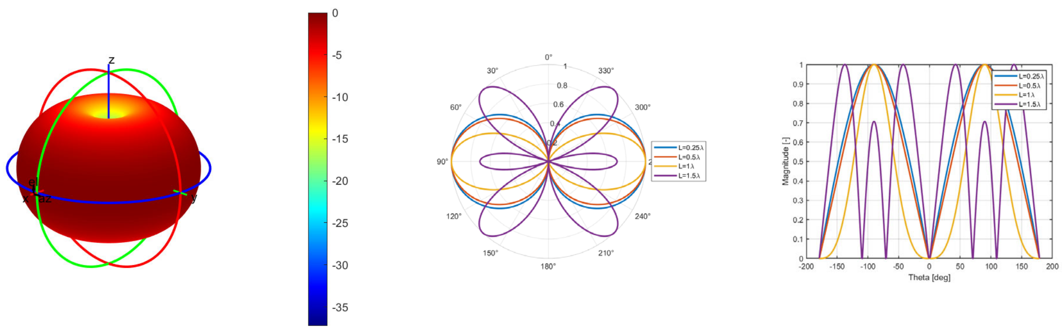
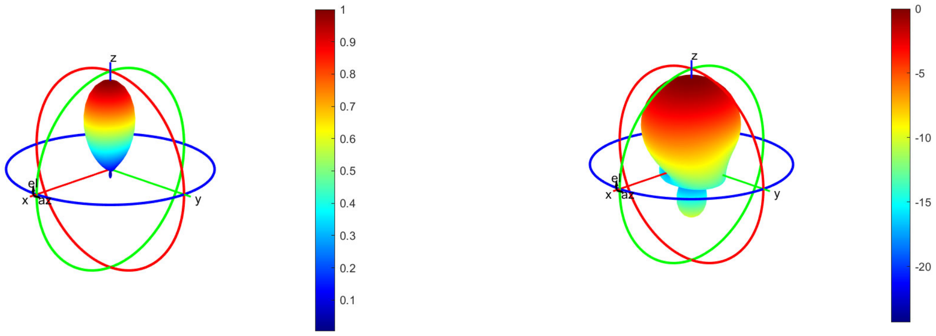
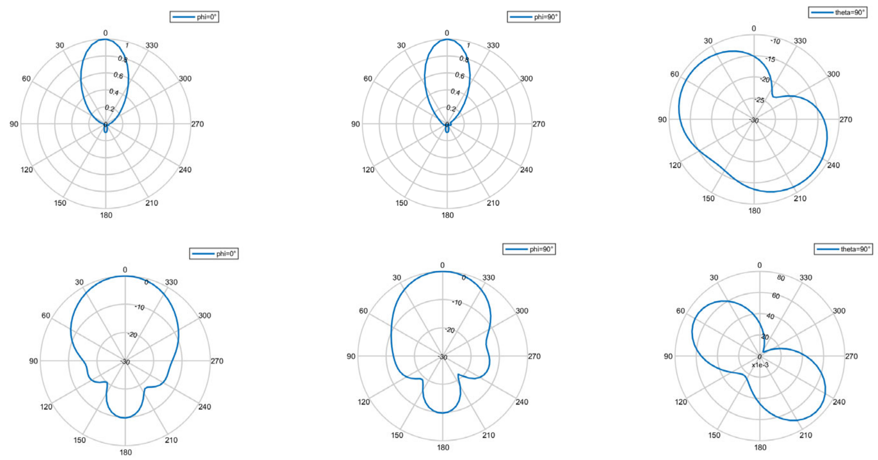
Exemple : omnidirectional pattern: dipole



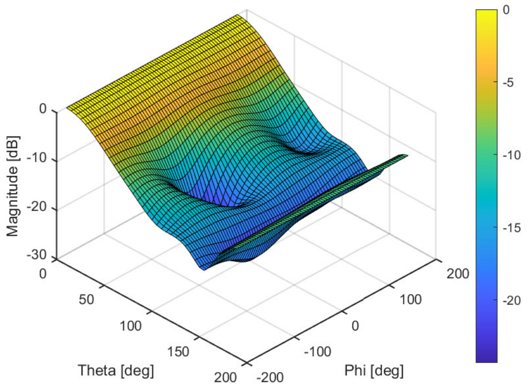
Example : directive antenna (horn antenna)



Example : directive antenna (horn antenna)



Example : directive antenna (horn antenna)

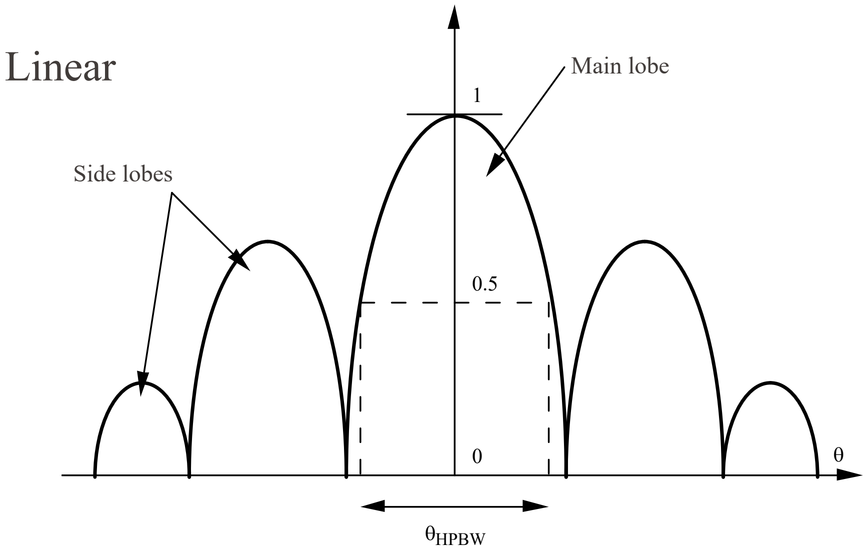


Definitions

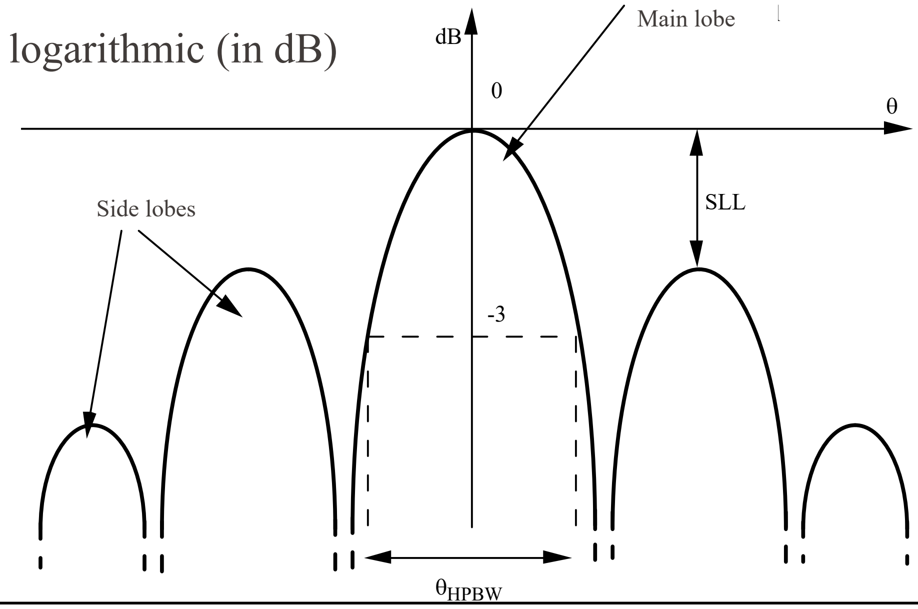
θ_{BW} : Beamwidth
θ_{HPBW}: Half Power Beam width

SLL = 10 log [P_{max} (main lobe) / P_{max} (sidelobes)]
Usually, P_{max} (main lobe) = 1
Thus: SLL = - 10 log [P_{max} (side lobes)]

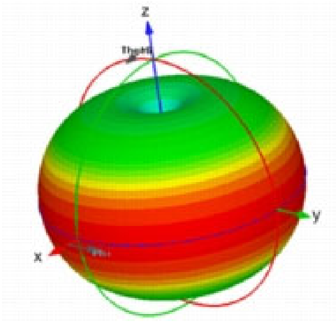
Radiation pattern



Radiation pattern



Radiated power and directivity



Radiated power: integral of p(r,θ,φ) on a spherical surface of radius r : ds = $r^2 \sin\theta$

$$P_{rad} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta r^2 \sin\theta\, p(r,\theta,\varphi) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta\, U(\theta,\varphi)$$

$$\langle p \rangle = p_{iso} = \frac{P_{rad}}{4\pi r^2}$$

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Directivity

$$D(\theta,\varphi) = \frac{p(r,\theta,\varphi)}{p_{iso}} = \frac{4\pi r^2 p(r,\theta,\varphi)}{P_{rad}} = \frac{4\pi r^2 p(r,\theta,\varphi)}{\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta r^2 \sin\theta\, p(r,\theta,\varphi)}$$

$$D(\theta,\varphi)_{dB} = 10\log_{10} D(\theta,\varphi)$$

$$D_{\max} = \max\left[D(\theta,\varphi)\right] = D(\theta_{\max},\varphi_{\max}) \geq 1$$

$$D_{\max dB} = 10\log_{10} D_{\max} \geq 0dB$$

$$\langle D(\theta,\varphi) \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta\, D(\theta,\varphi) = 1$$

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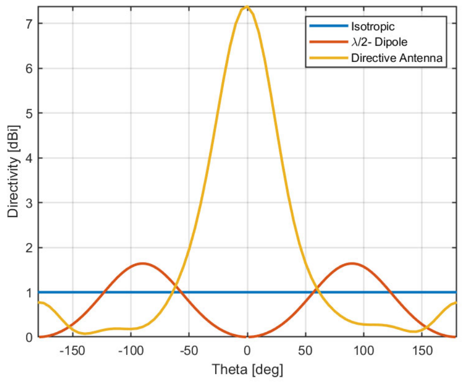
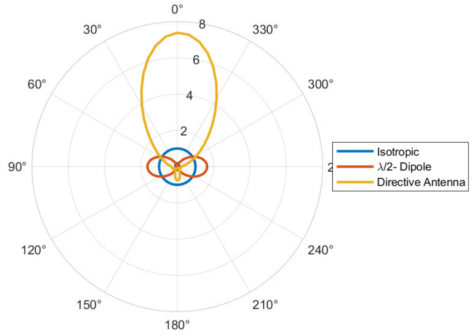
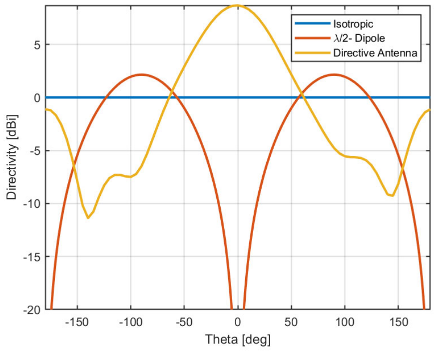
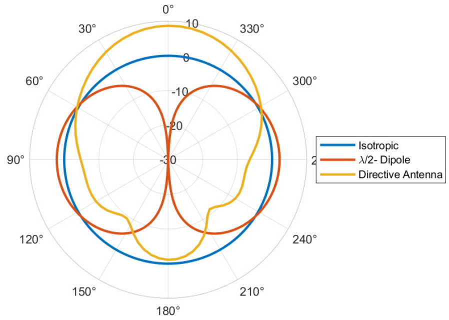
Directivity (linear)





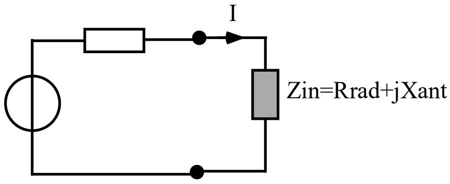
Directivity (dB)





Radiation resistance

For a transmitting antenna, we can define the following equivalent circuit



Power delivered to the antenna:

$$P_f = \frac{I^2}{2}\mathrm{Re}[Z_{in}]$$

Thus $$R_{rad} = \mathrm{Re}[Z_{in}] = 2P_f / I^2$$

Radiation resistance

- An antenna is a passive, linear and dissipating component
- Its impedance is given by Z_{in}
- The power delivered to the antenna is $P_f = I^2/2\ Re(Z_{in})$.
- In the absence of losses, we have, $P_f = P_{rad}$
- The radiation resistance is defined as $R_{rad} = Re(Z_{in}) = 2P_{rad} / I^2$
- X_{ant} is much harder to get (linked to reactive energy, usually obtained numerically)

Example : Hertzian dipole

Reminder

$$E_\theta(r,\theta,\varphi) = \frac{jZ_c}{2\lambda} I\Delta l \frac{e^{-jkr}}{r} \sin\theta$$

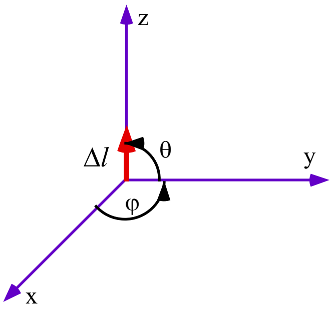
$$E_\varphi(r,\theta,\varphi) = 0$$

$$H_\theta(r,\theta,\varphi) = 0$$

$$H_\varphi(r,\theta,\varphi) = \frac{j}{2\lambda} I\Delta l \frac{e^{-jkr}}{r} \sin\theta$$

$$U(\theta,\varphi) = \frac{Z_c}{8\lambda^2}(I\Delta l)^2 \sin^2\theta$$

$$p(r,\theta,\varphi) = \frac{1}{r^2}\frac{Z_c}{8\lambda^2}(I\Delta l)^2 \sin^2\theta$$

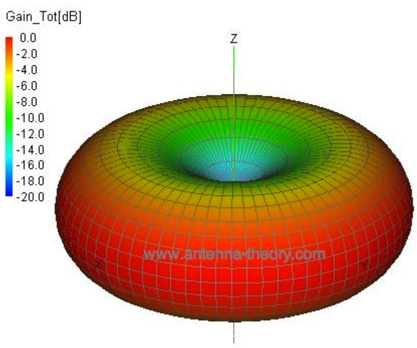


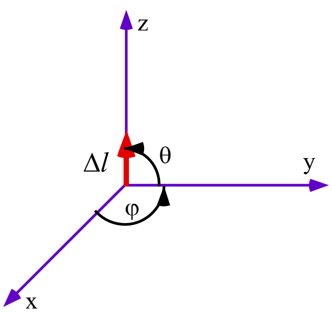
Example : Hertzian dipole

Patterns independ of φ

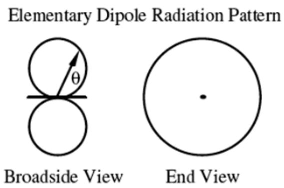
$$D_p(\theta) = \sin^2\theta$$

$$D_p(\theta)_{dB} = 10\log_{10}\sin^2\theta = 20\log_{10}\sin\theta$$





Example : Hertzian dipole



$$(\theta_{\max}) = 90°$$
$$D_p(\theta_{\max}) = 1$$

$$\sin^2\theta = 0 \quad \text{for } \theta = 0, 180°$$
$$\theta_{BW} = 180°$$
$$\sin^2\theta = 0.5 \text{ for } \theta = 45°,\ 135°$$
$$\theta_{HPBW} = 90°$$

Example : Hertzian dipole

$$P_{rad} = 2\pi\int_0^{\pi} d\theta r^2 \sin\theta\, p(r,\theta,\varphi) = \frac{2\pi}{3} Z_c I^2 (\Delta l/\lambda)^2$$

$$D(\theta,\varphi) = \frac{3}{2}\sin^2\theta \qquad D_{max} = 1.5 = 1.76 \text{ dB}$$

$$R_{rad} = \frac{2\pi}{3} Z_c (\Delta l/\lambda)^2 \approx 800(\Delta l/\lambda)^2\,[\Omega]$$

For a given length, the radiation resistance is proportional to the square of the frequency. For instance, a conductor of one meter will have a radiation resistance of 0.0088 Ω at 1 MHz, 0.88 Ω at 10 MHz, and 88 Ω at 100 MHz

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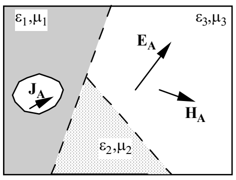
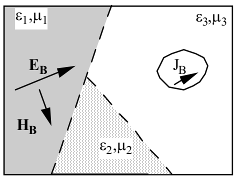
Classic reciprocity theorem applied to the antenna case





$$\mathbf{X} = \mathbf{E_A} \times \mathbf{H_B} - \mathbf{E_B} \times \mathbf{H_A} \qquad \nabla \bullet \mathbf{X} = \mathbf{J_A} \bullet \mathbf{E_B} - \mathbf{J_B} \bullet \mathbf{E_A}$$

$$\int_s \mathbf{ds} \bullet (\mathbf{E_A} \times \mathbf{H_B} - \mathbf{E_B} \times \mathbf{H_A}) = \int_v dv\, (\mathbf{J_A} \bullet \mathbf{E_B} - \mathbf{J_B} \bullet \mathbf{E_A})$$

Thanks to the divergence theorem!!

V: arbitrary volume, s: surface around V

Particular case: v-> infinity

$$\underbrace{\int_{v_A} d\mathbf{s}\left(\mathbf{E}_A\times\mathbf{H}_B-\mathbf{E}_B\times\mathbf{H}_A\right)}_{=0}=\int_{v_B} dv\left(\mathbf{J}_A\cdot\mathbf{E}_B-\mathbf{J}_B\cdot\mathbf{E}_A\right)$$

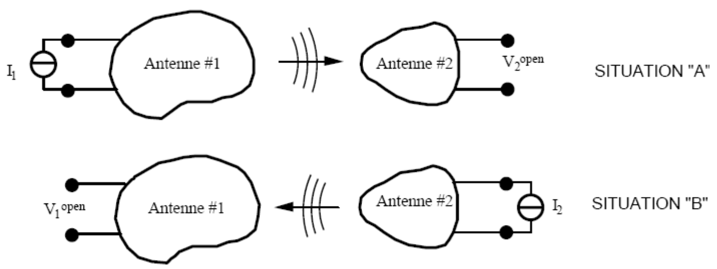
As we are in the far field and

$$\mathbf{H}=\frac{1}{Z_c}\hat{\mathbf{r}}\times\mathbf{E}$$

$$\mathbf{E}=Z_c\hat{\mathbf{r}}\times\mathbf{H}$$

$$\int_{v_A} dv\left(\mathbf{J}_A\cdot\mathbf{E}_B\right)=\int_{v_B} dv\left(\mathbf{J}_B\cdot\mathbf{E}_A\right)$$

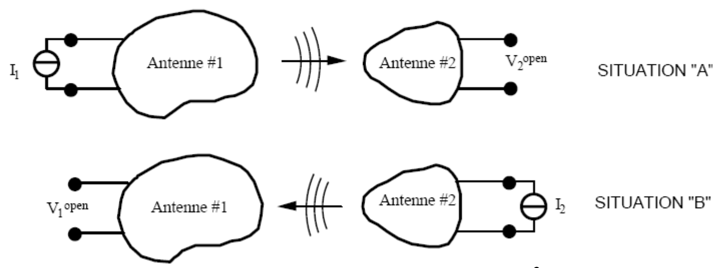
Reciprocity in circuit terms



"A" : #1 is the transmitter, connected to an ideal current source I_1.
#2 is the receiver, open-circuited and having a voltage V_2^{open}.
"B" : #2 is the transmitter, connected to an ideal current source I_2.
#1 is the receiver, open-circuited and having a voltage V_1^{open}.

EPFL

Reciprocity in circuit terms



$$\int_{v_A} dv\left(\boldsymbol{J}_A \cdot \boldsymbol{E}_B\right) = \int_{v_B} dv\left(\boldsymbol{J}_B \cdot \boldsymbol{E}_A\right)$$

Sources are external to the antennas, thus

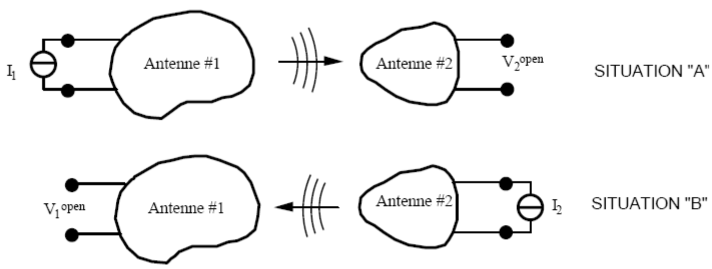
$dv = s\,dl$, $\boldsymbol{J}_A = \boldsymbol{l}(I_1/s)$ and $\boldsymbol{J}_A dv = I_1 \boldsymbol{dl}$

$$\int_{v_A} dv\left(\boldsymbol{J}_A \cdot \boldsymbol{E}_B\right) = I_1 \underbrace{\int_{L_A} dl\left(\boldsymbol{J}_B \cdot \boldsymbol{E}_A\right)}_{V_1^{open}(I_2)}$$

Thus $\quad I_1 V_1^{open}\left(I_2\right) = I_2 V_2^{open}\left(I_1\right)$

EPFL

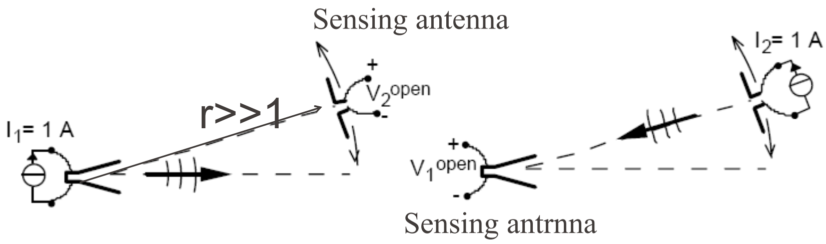
Reciprocity in circuit terms



Thus $\quad I_1 V_1^{open}\left(I_2\right) = I_2 V_2^{open}\left(I_1\right)$

andy $\quad V_1^{open}\left(I_2 = 1A\right) = V_2^{open}\left(I_1 = 1A\right)$

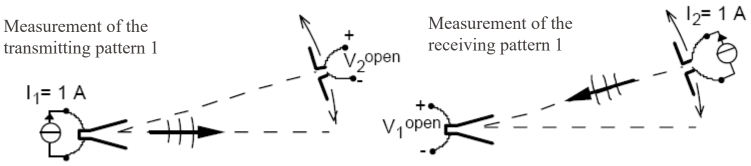
Transmitting and receiving patterns



The antenna pattern of the antenna in receiving mode is identical to the pattern in transmitting mode. This is a direct consequence of

$$I_1 V_1^{open}(I_2) = I_2 V_2^{open}(I_1)$$

Equivalent circuit of a transceiving system



$$V_1 = z_{11} I_1 + z_{12} I_2$$

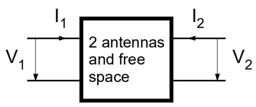
$$V_2 = z_{21} I_1 + z_{22} I_2$$

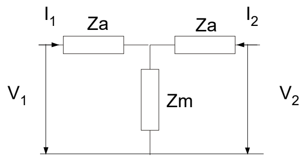
$$z_{12} = \left[\frac{V_1}{I_2}\right]_{I_1=0} = \frac{V_1^{open}}{I_2} \quad \text{and} \quad z_{21} = \frac{V_2^{open}}{I_1}$$

Reciprocity : $z_{12}=z_{21}$

"Privileged directions are the same at transmission and reception"

Equivalent circuit of a transceiving system





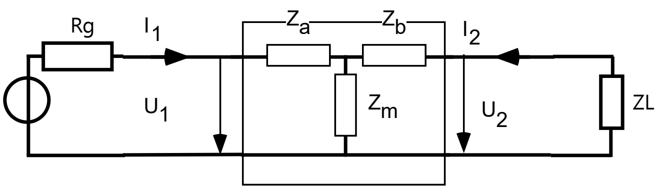
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix}$$

$$Z_a = Z_{11} - Z_{12}$$
$$Z_b = Z_{22} - Z_{12}$$
$$Z_m = Z_{12}$$

$Z_{11} = Z_{21}$, as

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_1^{open}}{I_2} \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{V_2^{open}}{I_1}$$

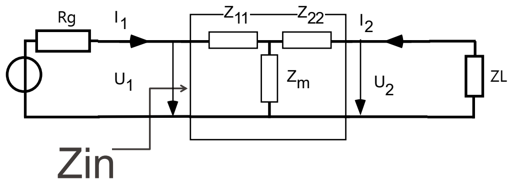
Equivalent circuit of a transceiving system



$$Z_a = Z_{11} - Z_m$$
$$Z_b = Z_{22} - Z_m$$

$$Z_{in,1} = Z_{11} - Z_m + Z_m \frac{Z_{22} - Z_m + Z_L}{Z_{22} + Z_L}$$

Equivalent circuit of a transceiving system



Consider that antenna #1 is transmitting, and is excited by a voltage generator U
Having an internal impedance Z_g. Antenna #2 is the receiving antenna loaded by Z_L

$$Z_{in,1} = Z_{11} - Z_m + Z_m \frac{Z_{22} - Z_m + Z_L}{Z_{22} + Z_L}$$

Annoying: the second antenna has an influence on the first

Antennas

Unilateral hypothesis

We suppose that the receiving antenna has no influence on the fields transmitted by antenna 1

Thus : The term $Z_{12}I_2$ is neglected

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{21}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \quad \text{becomes} \quad \begin{aligned} V_1 &= z_{11}I_1 \\ V_2 &= z_{21}I_1 + z_{22}I_2 = U_T + z_{22}I_2 \end{aligned}$$

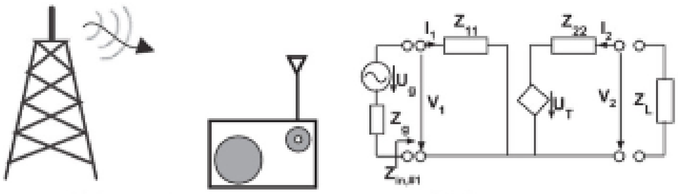
Where U_T is a dependent voltage generator

In this case, $Z_{in,\#1}=Z_{11}$

Antennas

Unilateral hypothesis

If the receiving antenna has no infleunce on the transmitting antenna

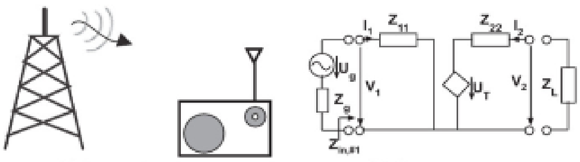


(a) Schéma du système (b) Circuit équivalent

$$V_1 = z_{11} I_1$$
$$V_2 = z_{21} I_1 + z_{22} I_2 = U_T + z_{22} I_2$$

In this case, $Z_{in,\#1} = Z_{11}$

Power reciprocity, under the unilateral hypothesis



(a) Schéma du système (b) Circuit équivalent

Power radiated by antenna 1:

$$P_{rad-1} = \frac{|U|^2}{4\,\mathrm{Re}[Z_{11}]}$$

Power available at the receiver of antenna 2:

$$P_{av-r2} = \frac{|z_m I_1|^2}{4\,\mathrm{Re}[Z_2]}$$

In the case of the unilateral hypothesis, there is a clear difference between transmission and reception

Power reciprocity, under the unilateral hypothesis

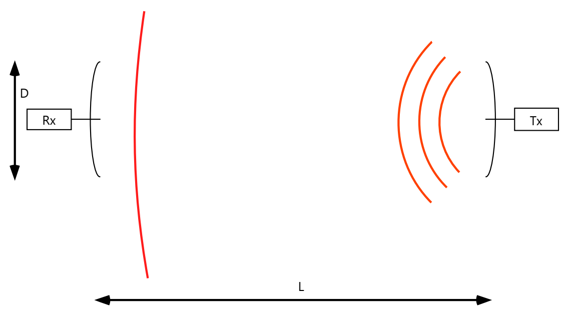
$$P_{rad-1} = \frac{|U|^2}{4\,\mathrm{Re}[Z_{11}]}$$ Power radiated by antenna 1

$$P_{av-r2} = \frac{|z_m I_1|^2}{4\,\mathrm{Re}[Z_2]}$$ Power available at the charge of antenna 2

$$|U| = 2\,\mathrm{Re}[z_{11}]\,|I\,|$$ By definition, thus

$$\frac{P_{av-r2}}{P_{rad-1}} = \frac{|z_m|^2}{4\,\mathrm{Re}[z_{11}]\,\mathrm{Re}[z_{22}]}$$ and $$\frac{P_{av-r2}}{P_{rad-1}} = \frac{P_{av-r1}}{P_{rad-2}}$$

Antenna effective aperture



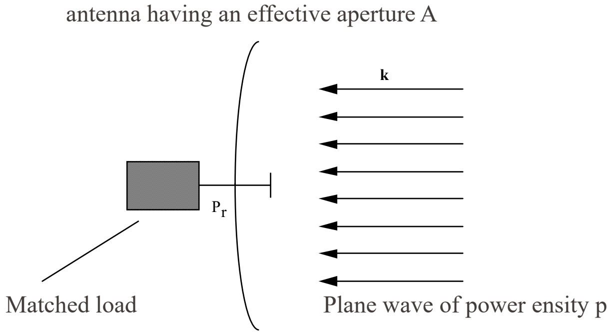
When *L>>>D*, the spherical wave transmitted can be seen as a plane wave at the receiving antenna. This wave has a power density *p*

Antenna effective aperture

The maximum available received power ($P_{av\text{-}r}$) is the power supplied by the receiving antenna when connected to a load impedance equal to the complex conjugate value of the antenna's internal impedance ($Z_{load} = Z_{in}^*$, conjugate matching). The effective aperture (A_e) of an antenna at reception is then defined as the ratio between the maximum available reception power and the incident power density.

$$A_e(\theta,\varphi) = \frac{P_{av-r}}{p} \left[m^2\right]$$

Antenna effective aperture



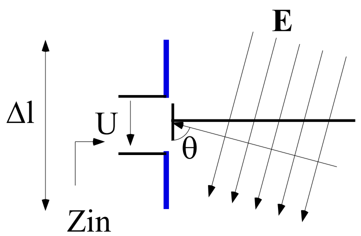
$$P_r = p\ A = \frac{E_0^2}{Z_c} A$$ Maximal received power

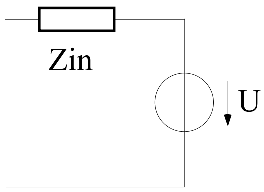
$$P_r(\theta,\varphi) = A_e(\theta,\varphi) p$$ Real received power

with

$$A_e(\theta,\varphi) < A$$

Example Hertzian dipole





Power density

$$p = \frac{|\mathbf{E}|^2}{Z_c}$$

$$U = |\mathbf{E}|\Delta l \sin\theta = \sqrt{Z_c p}\Delta l \sin\theta$$

$$P_{av-r} = \frac{U^2}{4\,\mathrm{Re}[Z_{in}]}$$

$$A_e(\theta) = \frac{P_{av-r}}{p} = \frac{U^2}{p}\frac{1}{4\,\mathrm{Re}[Z_{in}]}$$

We suppose

$$\mathrm{Re}[Z_{in}] = R_{rad} = \frac{2\pi}{3} Z_c (\Delta l/\lambda)^2$$

We get

$$A_e = \frac{3\lambda^2 \sin^2\theta}{8\pi}$$

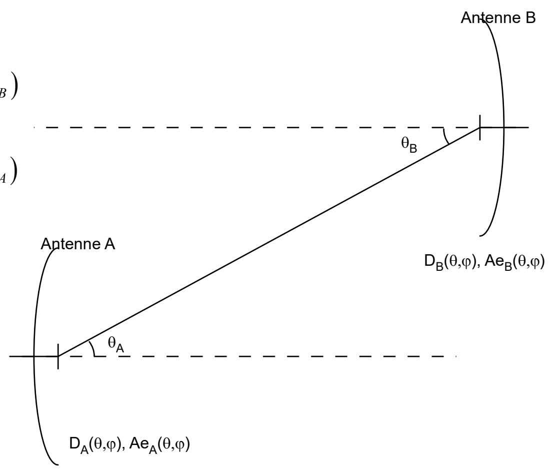
Link berween directivity and effective aperture

$$\left.\frac{P_r}{P_{fourni}}\right|_{A\to B} = \frac{1}{4\pi L^2} d_A(\theta_A,\varphi_A) A_{eB}(\theta_B,\varphi_B)$$

$$\left.\frac{P_r}{P_{fourni}}\right|_{B\to A} = \frac{1}{4\pi L^2} d_B(\theta_B,\varphi_B) A_{eA}(\theta_A,\varphi_A)$$

$$\frac{d_A(\theta_A,\varphi_A)}{A_{eA}(\theta_A,\varphi_A)} = \frac{d_B(\theta_B,\varphi_B)}{A_{eB}(\theta_B,\varphi_B)}$$

$$d(\theta,\varphi) = \frac{4\pi}{\lambda^2} A_e(\theta,\varphi)$$



Using the known values for the Hertzian dipole

Friis' formula for ideal systems

$$P_{av-rA} = P_{radB} d_A d_B \left(\frac{\lambda}{4\pi r} \right)^2$$

Antennas

Real antennas: what can go wrong ?

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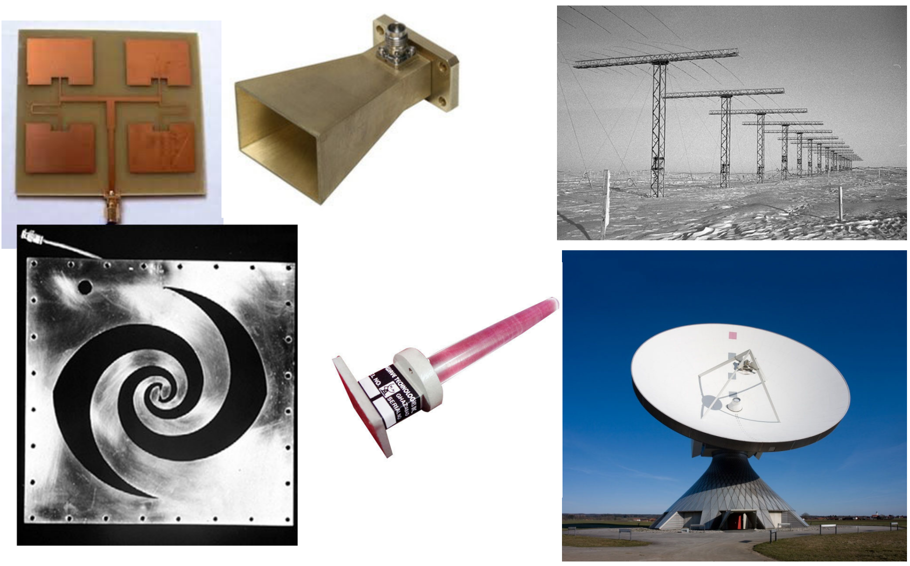
What are the difference between ideal antennas and reality ?

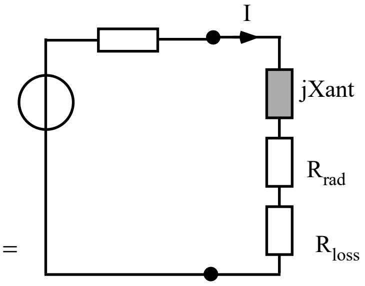
- Losses
- Impedance mismatch
- Polarization mismatch

Losses: conductive or dielctric losses in the materials



Real antenna: the efficiency

$$Z_{in} = R_{loss} + R_{rad} + jX_{ant}$$



Radiated power $P_{rad} = I^2 R_{rad}$

Available power $P_{av-e} = I^2 \,\mathrm{Re}[Z_{in}] = I^2 (R_{rad} + R_{loss})$

Antenna efficiency $$\eta = \frac{P_{rad}}{P_{av-e}} = \frac{R_{rad}}{R_{rad} + R_{loss}} \le 1$$

The antenan gain

The gain is defined as

$$G(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{av-e}}$$

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{rad}}$$

Thus $$\frac{G(\theta,\varphi)}{D(\theta,\varphi)} = \eta \le 1$$

Typical antenna efficiencies

- Around 100% for a horn, a wire antenna or a parabola
- 10-60% for a smartphone antenna
- around 1% for a radio antenna on a car

Example: The Hertzian dipole

Radiated power

$$P_{rad} = 2\pi\int_0^\pi d\theta r^2 \sin\theta\, p(r,\theta,\varphi) = \frac{2\pi}{3} Z_c I^2 (\Delta l/\lambda)^2$$

Radiation resistance

$$R_{rad} = \frac{2\pi}{3} Z_c (\Delta l/\lambda)^2 \approx 800(\Delta l/\lambda)^2 [\Omega]$$

Ohmic losses is a conductive wire of lenght Δl
And radius a (skin effect)

$$R_{loss} = \frac{\Delta l}{2\pi a}\sqrt{\frac{\pi\mu f}{\sigma}}$$

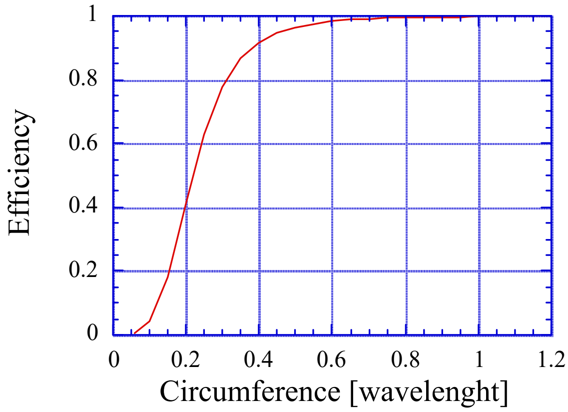
efficiency $$\eta = \frac{P_{rad}}{P_{av-e}} = \frac{R_{rad}}{R_{rad} + R_{loss}} \le 1$$

EPFL Example: The short dipole

Δl=1m, σ= 5.7 10^7 (cuivre)

f	R_{rad} [Ω]	R_{loss}[Ω]	η
1 MHz	0.0088	0.1	8 %
10 MHz	0.88	0.3	73 %
100 MHz	88.	1.	98.9 %

EPFL Efficiency



EPFL Impedance mismatch

$$P_{av-e} = I^2 \,\mathrm{Re}[Z_{in}]$$

$P_{rad} = P_{av-e}$ Without losses $\qquad$ $P_{rad} = \eta P_{av-e}$ With losses

$$R_{rad} = \frac{P_{rad}}{I^2}$$

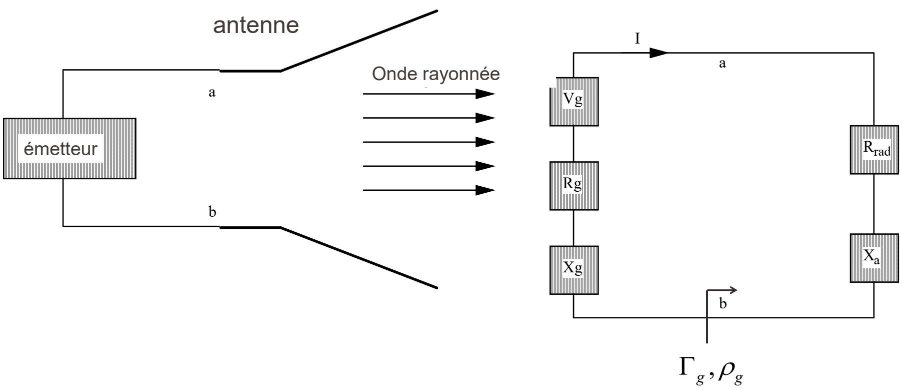
The power transfer to the antenna is maximum, and therefore the radiated power is maximum when the antenna is matched to the generator: $Z_{in} = Z_g^*$

In general, the impedance of the generator is real: R_g

In this case, the max poawer is

$$P_{av-e} = \frac{1}{2} V_g I_g^* = \frac{|V_g|^2}{4R_g}$$

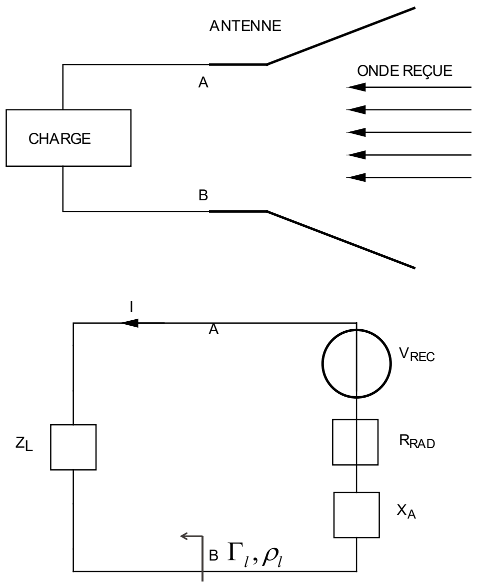
EPFL Mismatch



The power delivered to the antenna is not always the available power $P_{\text{av-e}}$

$$\rho_g = \frac{Z_{in} - Z_g}{Z_{in} + Z_g}$$

Mismatch



$$\rho_l = \frac{Z_l - Z_{in}}{Z_l + Z_{in}}$$

The power delivered to the load is not always the power available at the antenna

$Pr_{\text{load}} = \eta P_{\text{av-r}}$

Mismatch

If there is a mismatch between the antenna and the generator or the load (reception), the power supplied is given by

$$P_f = P_{av-e}\left(1-\rho_g^2\right) \qquad \rho_g = \frac{Z_{in} - Z_g}{Z_{in} + Z_g}$$

And the power supplied by the antenna to load by

$$\mathrm{P}_{r_{load}} = P_{av-r}\left(1-\rho_l^2\right) \qquad \rho_l = \frac{Z_l - Z_{in}}{Z_l + Z_{in}}$$

where ρ_g is the reflection coefficient between the generator and the antenna and ρ_l the reflection coefficient between the antenna and the load

Friis's formula

Ideal case $P_{r_{load}} = P_{av-e} D_r D_{tr} \left(\frac{\lambda}{4\pi R} \right)^2$

Mismatched antennas: $P_{r_{load}} = P_{av-e} D_r D_{tr} \left(1-\rho_r^2\right)\left(1-\rho_{tr}^2\right)\left(\frac{\lambda}{4\pi R} \right)^2$

With losses $P_{r_{load}} = P_{av-e} D_r D_{tr} \eta_r \eta_{tr} \left(1-\rho_r^2\right)\left(1-\rho_{tr}^2\right)\left(\frac{\lambda}{4\pi R} \right)^2$

$$= P_{av-e} G_r G_{tr} \left(1-\rho_r^2\right)\left(1-\rho_{tr}^2\right)\left(\frac{\lambda}{4\pi R} \right)^2$$

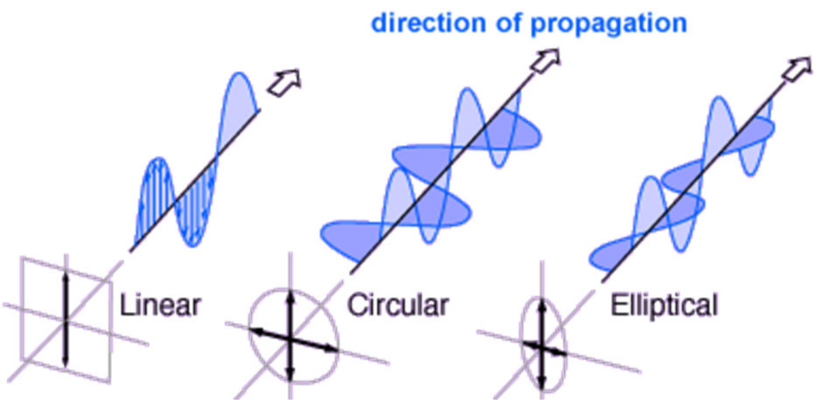
Real antennas

Transmitting $P_{rad} = \eta_1 P_f = \eta_1 \left(1 - \left|\rho_g\right|^2\right) P_{av-e}$

Receiving $P_{r_{load}} = \eta_1 P_r = \eta_1 \left(1 - \left|\rho_g\right|^2\right) P_{av-e}$

Polarization

Polarisation



Jairam Sankar, https://www.electronicsforu.com/resources/learn-electronics/antenna-polarisation

Polarisation

For a monochromatic wave, we have:

$$\boldsymbol{E}(t) = \sqrt{2}\left[\hat{\mathbf{x}}E_{0x}\cos(\omega t + \varphi_x) + \hat{\mathbf{y}}E_{0y}\cos(\omega t + \varphi_y) + \hat{\mathbf{z}}E_{0z}\cos(\omega t + \varphi_z)\ \right]$$

Polarization

$$\boldsymbol{E}(t) = \sqrt{2}\left[\hat{\mathbf{x}}E_{0x}\cos(\omega t + \varphi_x) + \hat{\mathbf{y}}E_{0y}\cos(\omega t + \varphi_y) + \hat{\mathbf{z}}E_{0z}\cos(\omega t + \varphi_z) + \right]$$

$$\mathbf{E}(t) = \mathbf{E}(0)\cos(\omega t) + \mathbf{E}(T/4)\sin(\omega t)$$

$$\boldsymbol{E}(0) = \sqrt{2}\left[\hat{\mathbf{x}}E_{0x}\cos(\varphi_x) + \hat{\mathbf{y}}E_{0y}\cos(\varphi_y) + \hat{\mathbf{z}}E_{0z}\cos(\varphi_z)\right]$$

$$\boldsymbol{E}(T/4) = -\sqrt{2}\left[\hat{\mathbf{x}}E_{0x}\sin(\varphi_x) + \hat{\mathbf{y}}E_{0y}\sin(\varphi_y) + \hat{\mathbf{z}}E_{0z}\sin(\varphi_z)\right]$$

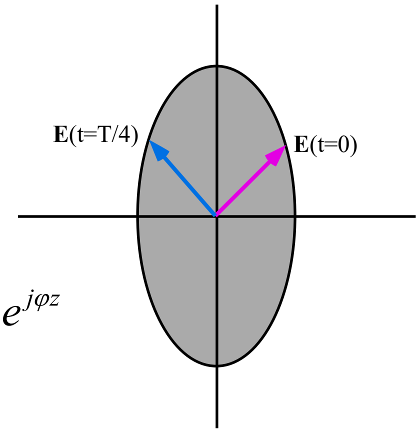
Polarization

In terms of phasors:

$$\mathbf{E}(0) = \mathrm{Re}[\mathbf{E}]$$

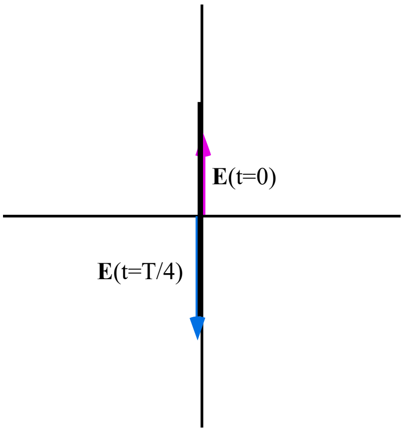
$$\mathbf{E}(T/4) = -\mathrm{Im}[\mathbf{E}]$$

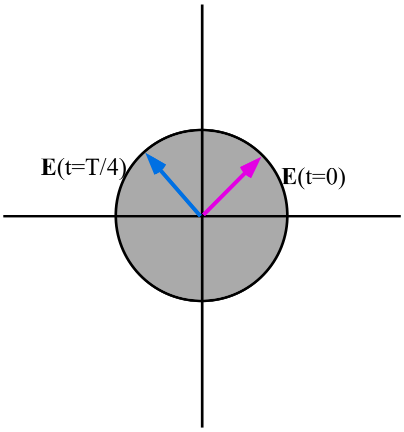
$$\mathbf{E} = \hat{\mathbf{x}} E_{0x} e^{j\varphi_x} + \hat{\mathbf{y}} E_{0y} e^{j\varphi_y} + \hat{\mathbf{z}} E_{0z} e^{j\varphi z}$$



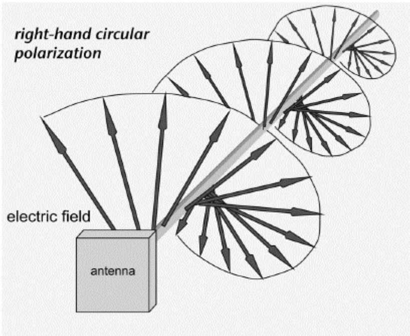
$$\boldsymbol{E}(t) = E(0)\cos(\omega t) + E(T/4)\sin(\omega t)$$

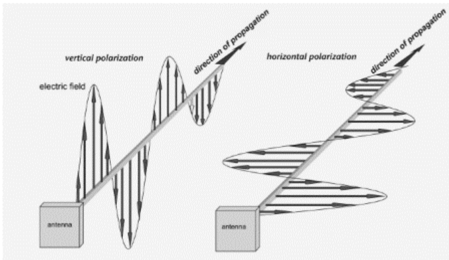
Special cases





Special cases





https://www.electronicsforu.com/resources/learn-electronics/antenna-polarisation

Special cases

Linear polarization:

$$\boldsymbol{E}(0)\times\boldsymbol{E}(T/4)=0$$

$$\langle E^2\rangle\neq 0$$

Circular polarization:

$$\boldsymbol{E}(0)\cdot\boldsymbol{E}(T/4)=0$$

$$|\boldsymbol{E}(0)|=|\boldsymbol{E}(T/4)|\neq 0$$

$$\mathbf{E}\cdot\mathbf{E}=0$$

Friis' formula

$$\frac{P_{r_{load}}}{P_{av-e}} = G_e G_r \left(\frac{\lambda}{4\pi r}\right)^2 \left|\boldsymbol{e}_e \cdot \boldsymbol{e}_r^*\right|^2$$

Where $\mathbf{e}_e$ ($\mathbf{e}_r$) is the polarization vector of the transmitting (receiving) antenna. This vector is a unit vector that represents the polarization of the wave emitted by this antenna.

$\left|\boldsymbol{e}_e \cdot \boldsymbol{e}_r^*\right|^2$ Is the depolarization factor

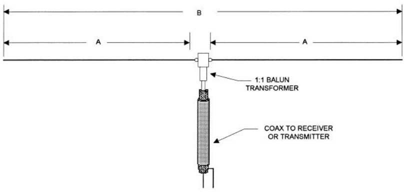
Friis' formula for a real transmission

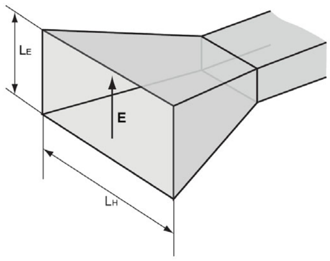
In free space $$P_{r_{load}} = P_{av-e} G_r G_{tr} \left(1-\rho_g^2\right)\left(1-\rho_l^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 \left|\boldsymbol{e}_e \bullet \boldsymbol{e}_r^*\right|^2$$

Depolarization factor $$\chi_{pol} = \left|\boldsymbol{e}_e \bullet \boldsymbol{e}_r^*\right|^2$$

ρ_l: reflection coefficient at the receiving antenna
ρ_g: reflextion coefficient at the transmitting antenna
$\boldsymbol{e}_r$: polarization vector of the receiving antenna
$\boldsymbol{e}_e$: polarization vector of the transmitting antenna

examples: linear polarization





examples: circular polarization